Fraternities and Labor Market Outcomes

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Fraternities

- Fraternities are more than "club good":
  - too expensive
  - people mention them on resumes
- Fraternity affiliation has positive effect on expected wage.
- Firms and fraternities realize that.
- Fraternities conduct intensive screening of applicants.
We want to explain

- How people decide whether to pledge.
- How fraternities select students to admit.
- What are the implications of the outcome of the pledge game for the (expected) wages of students of different abilities.
There is an equilibrium where everybody wants to join.
There is an equilibrium where some people are accepted but do not apply.
   It is not the highest types who earn the most from signaling.
   It is lowest types who are admitted who earn the most.
   Biggest losers are lowest types who are not admitted.
   This is the empirical equilibrium, and fraternity membership is economically significant.
New labor market participants are *students*, mass 1. Each student can be represented as a pair $(\theta, \mu) \sim h(\cdot) > 0$.
- $\theta$ is student’s potential productivity after employment.
- $\mu$ is student’s socializing value.
- $\theta$ and $\mu$ are independent.

Students like money and socializing.
- The representative *fraternity* likes students with high $\mu$ and students with high expected wage; has limited capacity.
- *Firms* offer competitive wages:
  - firms observe club membership and a signal about productivity $\tilde{\theta} \sim f_{\tilde{\theta}}(\cdot|\theta)$;
  - wage is equal to expected $\theta$ conditional on observables.
The World

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Game Timing

1. Students, having beliefs about distribution of other students in a fraternity, decide whether it is profitable to join the fraternity.

2. The fraternity picks an admittance rule.

3. Some students become fraternity members; values of productivity signals are realized.

4. Firms, observing membership of students in fraternity, assign wages to combinations of \( \tilde{\theta} \) and membership status.

In a rational expectations equilibrium, everyone’s beliefs are consistent with actions of everyone.
Firm’s Problem

Each firm observes a continuum of students with pdf $h(\theta, \mu)$, has a common knowledge of signaling technology $f_{\tilde{\theta}}(\tilde{\theta}|\theta)$, and knows the distribution of students in (and out of) the fraternity

$$c(\theta, \mu) = \mathbb{I}(\theta, \mu \text{ is in the club})$$

Then the wage offered to a frat member with signal $\tilde{\theta}$ is

$$w_C(\tilde{\theta}) = \frac{\int \theta h(\theta, \mu) c(\theta, \mu) f_{\tilde{\theta}}(\tilde{\theta}|\theta) \, d\theta d\mu}{\int h(\theta, \mu) c(\theta, \mu) f_{\tilde{\theta}}(\tilde{\theta}|\theta) \, d\theta d\mu}$$
Student’s Problem

Students anticipate wages offered by firms, and possess a common knowledge about signaling technology $f_{\tilde{\theta}}(\tilde{\theta}|\theta)$.

Student $(\theta, \mu)$’s utility outside the fraternity is

$$U_{\overline{C}} = E_{\tilde{\theta}} \left[ w_{\overline{C}}(\tilde{\theta}) | \theta \right]$$

Student $(\theta, \mu)$’s utility inside the fraternity is

$$U_{C} = E_{\tilde{\theta}} \left[ w_{C}(\tilde{\theta}) | \theta \right] + n\mu - c$$

Students’ solution is:

$$\alpha(\theta, \mu) = I(U_{C} \geq U_{\overline{C}}|\theta, \mu) \quad A = ((\theta, \mu)|\alpha(\theta, \mu) = 1)$$
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$$U_C = E_\tilde{\theta} \left[ w_C(\tilde{\theta})|\theta \right] + n\mu - c$$

Students’ solution is:

$$a(\theta, \mu) = I(U_C \geq U_{\overline{C}}|\theta, \mu) \quad A = ((\theta, \mu)|a(\theta, \mu) = 1)$$
The Fraternity’s Problem

The fraternity observes set $A$ and anticipates same wage functions as students do, and picks set $B$ of admitted people. Club’s utility function is assumed to be

$$W(B) = W_1 \int_{A \cap B} E_{\tilde{\theta}} w_C(\tilde{\theta} | \theta) dH(\theta, \mu) + W_2 \int_{A \cap B} \mu dH(\theta, \mu)$$

s.t. $\int_{A \cap B} h(\theta, \mu) d\mu d\theta \leq \Gamma$

Here $\Gamma$ is a fraternity’s capacity constraint. Intersection of sets of wishing students $A$ and admitted students $B$ is the set $C$ — fraternity members.
### Cutoff Rules

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Cutoff Rules

Proposition

There is a cutoff $\mu_A(\theta)$ such that people with $\mu$ bigger than that pledge.

Proposition

There is a cutoff $\mu_B(\theta)$ such that people with $\mu$ bigger than that are admitted.

Proposition

If signaling technology has a MLRP property, $\mu_B(\theta)$ is decreasing in $\theta$. 
Fraternity’s Cutoff Rule

Applicant types accepted by Fraternity

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Need Imperfect But Good Signaling

**Proposition**

If signals $\tilde{\theta}$ are perfectly revealing, fraternity membership in equilibrium does not affect wages.

**Proposition**

If signals $\tilde{\theta}$ are useless, fraternity membership in equilibrium does not affect wages.
Assume \((\theta, \mu) \in [0, 1]^2\), and \((\theta, \mu)\) are uniformly distributed. Also assume that three productivity signals are possible: \(H\), \(M\) and \(L\).

\[
P(\tilde{\theta} = L|\theta) = 1 - 2\theta, \theta \in [0, \frac{1}{2}]
\]
\[
P(\tilde{\theta} = H|\theta) = 2\theta, \theta \in [\frac{1}{2}, 1]
\]

Then two classes of nontrivial equilibria can be observed.
Application-Unconstrained Equilibrium

Would like to join the club, but are not accepted

Club members

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Application-Constrained Equilibrium

Desired by club, but don’t apply

Club members

Would like to join the club, but are not accepted

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Wages Structure

Application–Constrained Equilibrium

Application–Unconstrained Equilibrium

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**Assumption**

Either the support for signals $\tilde{\theta}$ is finite, or the support of $f_{\tilde{\theta}}(\tilde{\theta}|\tilde{\theta})$ is non-trivial.

**Assumption**

The cost $c$ of joining the fraternity satisfies

$$n\mu + \bar{\theta} - E[\theta] < c < n\bar{\mu} + \bar{\theta} - E[\theta].$$

**Proposition**

Suppose that Assumptions 1 and 2 hold, and the fraternity is small enough, the equilibrium is single-peaked.
8634 GPAs of seniors in Fall of 2007.
701 GPAs of fraternity/sorority members in Fall of 2007.
Cannot match — no other info.

\[ P(\Phi|\text{GPA}) = P(\Phi) \frac{f_{\text{GPA}}(\text{GPA}|\Phi)}{f_{\text{GPA}}(\text{GPA})}. \]

Consistent estimates of \( f(\cdot) \) densities will yield consistent estimate of quantity of members conditional on GPA.
Data - UIUC Fraternities

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Estimation

- Treat quantiles of GPA as "true" ability.
- Take 20 equispaced points of $\theta$ and estimates of $P(\Phi|\theta)$.
- Use the three-signal model, fit pseudopoints into 2-kink cutoff line with OLS.
- Add the condition that the cutoff is consistent with the model.
OLS Estimation

Data

Club Cutoff

Students’ Cutoff

OLS estimate

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Penalized OLS Estimate

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### Parameters

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<th>Estimate</th>
<th>95% confidence</th>
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<tr>
<td>$n$</td>
<td>0.2771</td>
<td>(0.1193, 0.5312)</td>
</tr>
<tr>
<td>$c$</td>
<td>0.2281</td>
<td>(0.0895, 0.4449)</td>
</tr>
<tr>
<td>$c/n$</td>
<td>0.8234</td>
<td>(0.7141, 0.8147)</td>
</tr>
<tr>
<td>$W_1/W_2$</td>
<td>0.2227</td>
<td>(0.0565, 0.3346)</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>0.1563</td>
<td>(0.1546, 0.1577)</td>
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Welfare Implications

Comparison to No Fraternity situation

Those who benefit

Those who lose

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Welfare Implications

Comparison to No Wage Shift situation

Those who benefit

Those who lose

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Conclusion

1. Frat members earn higher (on average) wage than non-members.
2. ... not necessarily so when condition on true ability.
3. There are two types of equilibria:
   - application-constrained ("single-peaked");
   - application-unconstrained ("ΦBK").
4. Single-peaked equilibrium exists very generally.
5. We get single-peaked fraternity in estimates.
6. "Single-peaked" effect is damaging for highly-able member students...
7. ... damaging for low-able non-members...
8. ... beneficial for low-type members.